Magnetic Fields of Vortices in a Superconducting Thin Film

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Abstract

The magnetic fields associated with a single superconducting vortex traversing a thin film are calculated. The formulation of Pearl, which has been used for a geometry in which for z<0 one has a vacuum and for z>0 one has superconducting material, is extended to the case of a thin film. In the Pearl geometry the flux exiting the sample is less than a flux quantum, while deep in the superconductor it is a flux quantum. For the thin film, the flux even in the mid-plane, is less than a flux quantum, but becomes equal to it for thickness much greater than the penetration depth. The magnetic field near the surface in both geometries has a significant radial component. The fields for a vortex array are then obtained by summing the fields from nearby vortices. The measurability of the field distributions is discussed.

1. Introduction

With the possibility of using the muon spin rotation technique (μSR) for thin films it has become possible to study superconducting films with thickness comparable to the superconducting penetration depth (λ). Niedermayer et al.[1] have already used μSR to study the internal fields near the surface of a superconductor, but one for which the overall thickness was considerably larger than λ. Here we present a calculation of the fields produced by vortices in films for which the total superconducting material thickness is comparable to λ.

2. The Calculation

While we could have obtained the internal fields by summing appropriate pancake vortices, see Clem[2], we have followed J. Pearl in his treatment of a metal-air interface[3]. While he had a surface separating infinite metal and infinite air regions, we consider a thin film between two infinite air regions. Pearl used Ginzberg-Landau electrodynamics for the superconductor so the vector potential satisfies:

$$\nabla \times \nabla \times \mathbf{A} + (1/\lambda)\mathbf{A} = \frac{\phi_0 \hat{\theta}}{2\pi\lambda^2 r} \quad (1)$$

We are first interested in the field distribution of a vortex throughout a film of thickness comparable to the penetration depth. The primary differences between our calculation and that of Pearl are that we have two boundaries and that we require symmetry about the thin film's mid-plane. The boundary conditions are that the vector potential and its derivative are continuous across the boundary.

The functional form for the vector potential in Pearl's geometry, for which the material for $z > 0$ is superconducting and for $z < 0$ is vacuum, inside the superconductor is:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{\gamma^2 + 1/\lambda^2} \cdot \left(1 - \frac{\gamma \exp(-(\gamma^2+1/\lambda^2)^{1/2} z)}{\gamma + (\gamma^2+1/\lambda^2)^{1/2}}\right) d\gamma \quad (2)$$

Where $J_1(\gamma r)$ is a Bessel function. If we introduce $s = (\gamma^2 + 1/\lambda^2)^{1/2}$ this can be written as:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \left(1 - \frac{\gamma \exp(-sz)}{\gamma + s}\right) d\gamma \quad (3)$$

Outside the superconductor, $z < 0$, the solution is:

$$f_1 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} e^{\gamma z} \frac{s}{s+\gamma} d\gamma \quad (4)$$

The function f_1 is the solution of:

$$\frac{\partial^2 f_1}{\partial z^2} + \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} r f_1 = 0 \quad (5)$$

The function f_2 is the solution of:

$$\frac{\partial^2 f_2}{\partial z^2} + \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} r f_2 - \frac{1}{\lambda^2} f_2 = -\frac{\phi_0}{2\pi\lambda^2 r} \quad (6)$$

For a thin film of thickness d, the solution inside the superconductor is:

$$f_2 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \cdot \left(1 - \frac{\gamma[\exp(-s(z+d/2)) + \exp(+s(z-d/2)))]}{s(1-e^{-sd}) + \gamma(1+e^{-sd})}\right) d\gamma \quad (7)$$

The solution outside the superconductor and for $z > d/2$ is:

$$f_1 = \int_0^\infty \frac{\phi_0}{2\pi\lambda^2} \frac{J_1(\gamma r)}{s^2} \frac{(1-e^{-sd}) e^{\gamma(z+d/2)} s}{s(1-e^{-sd}) + (1+e^{-sd})\gamma} d\gamma \quad (8)$$

for $z < -d/2$. For $z > d/2$ the exponential is $e^{-\gamma(z-d/2)}$.

3. Results

The general shape of the fields may be seen in Fig. 1.

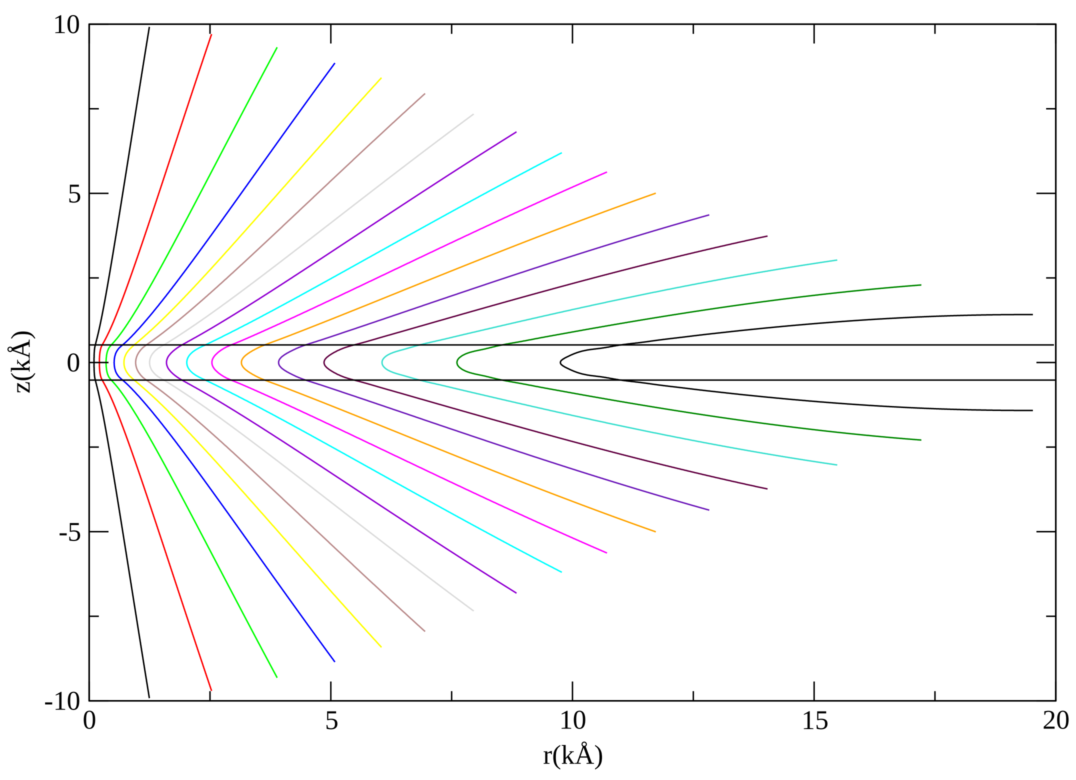

Figure 1: *Typical field lines for a thin film of 100 nm and λ=130 nm. One can see the radial component increasing as one is further from the vortex core and nearer the surface.*

The radial dependence of the field perpendicular component of the magnetic field near the film's surface is shown in Fig. 2. The radial dependence of the radial field component near the film's surface is shown in Fig. 3

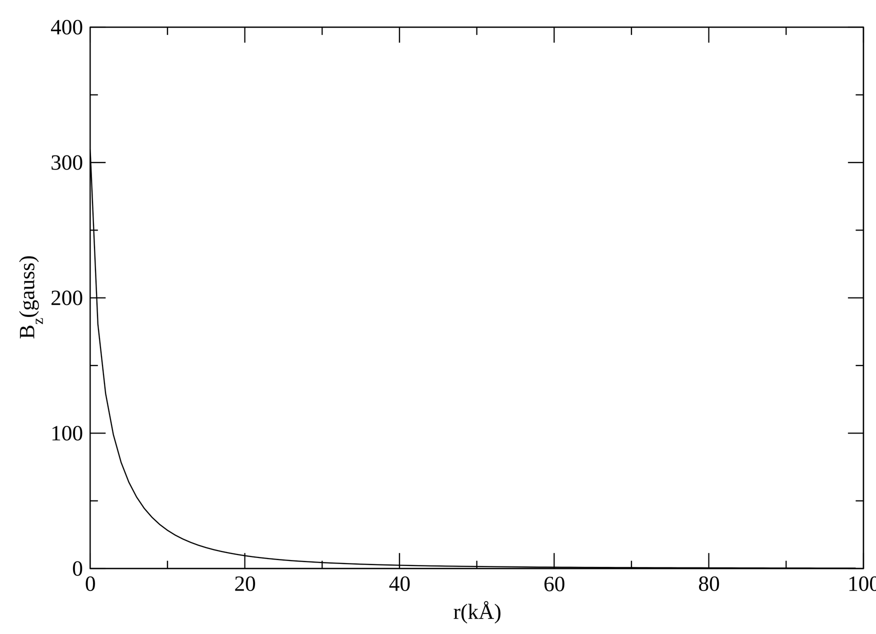

Figure 2: *The perpendicular component of the magnetic field for a single vortex as a function of radius from its core. This is for near the surface of the superconductor of thickness 100 nm and penetration depth 130 nm.*

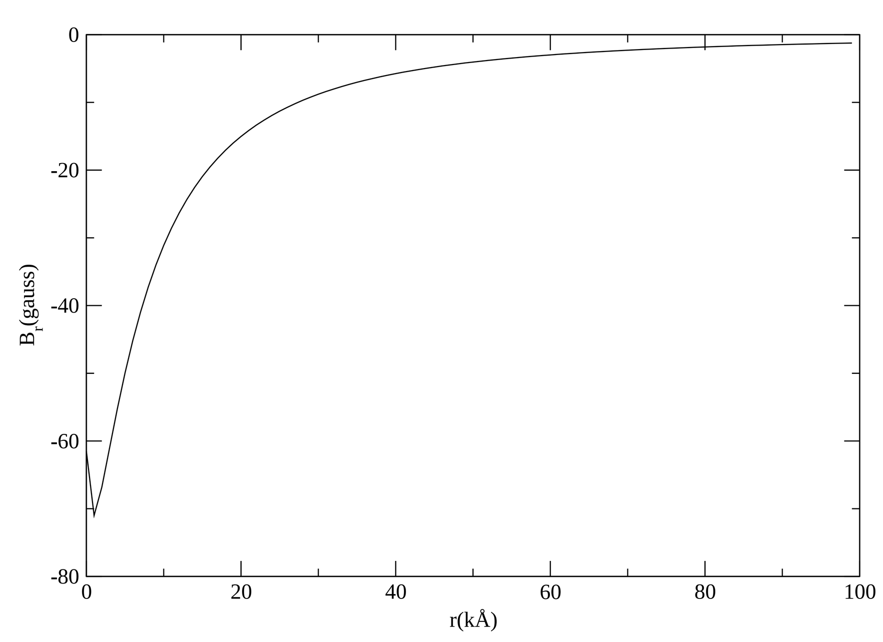

Figure 3: *The radial component of the magnetic field for a single vortex as a function of radius from its core. This is for near the surface of the superconductor of thickness 100 nm and penetration depth 130 nm.*

Perhaps the most surprising result is shown in Fig. 4. In this figure the flux for one vortex is shown as a function of film thickness, d, for various penetration depths. This flux is calculated in the mid-plane of the film where the magnetic fields are only in the **z** direction. All of these curves approach the flux quantum ϕ_0 for large thickness. However, in the regime where d is comparable to λ a considerable reduction in the flux occurs.

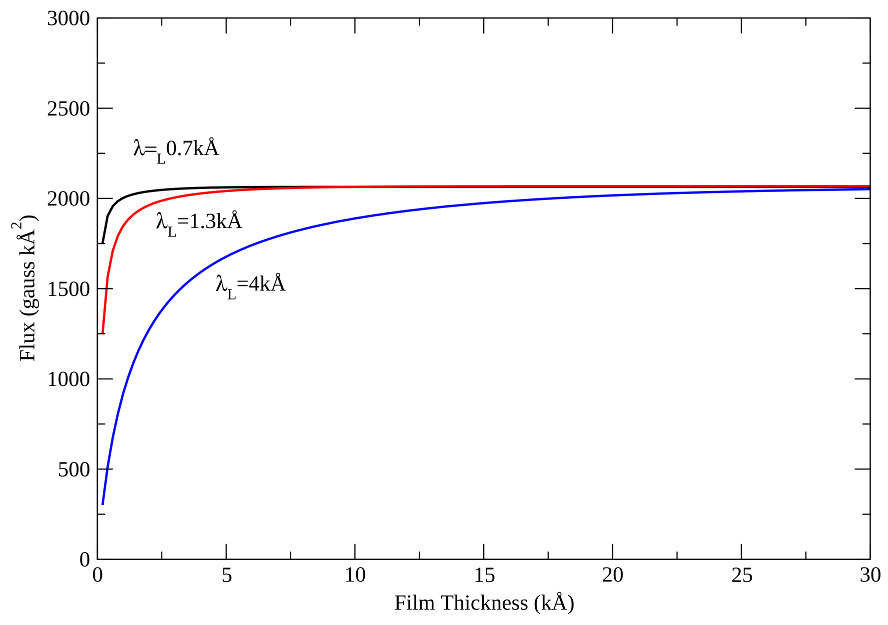

Figure 4: *The total magnetic flux in the mid-plane of the film as a function of film thickness, d, for various penetration depths, λ. All the curves approach ϕ_0 for thick films.*

We have also calculated the field distribution for a triangular array of vortices by summing the fields from individual vortices. We then used this field distribution to simulate μSR data. These are shown in Figs. 5 and 6.

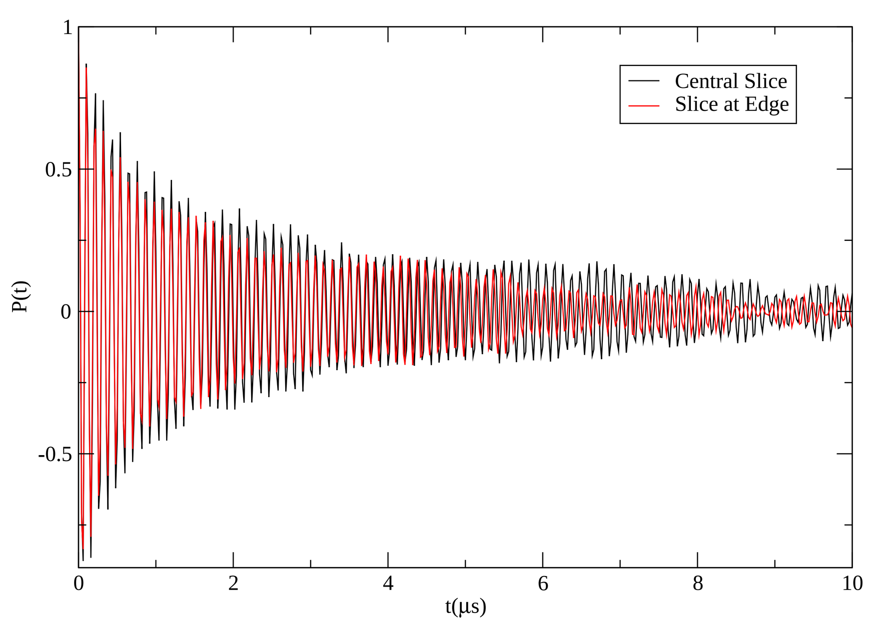

Figure 5: *Simulated μSR data for the initial muon polarization transverse to the magnetic field which would be perpendicular to the film. The results for two regions are shown, black: muons stopping near the surface of the film and red: stoping near the center.*

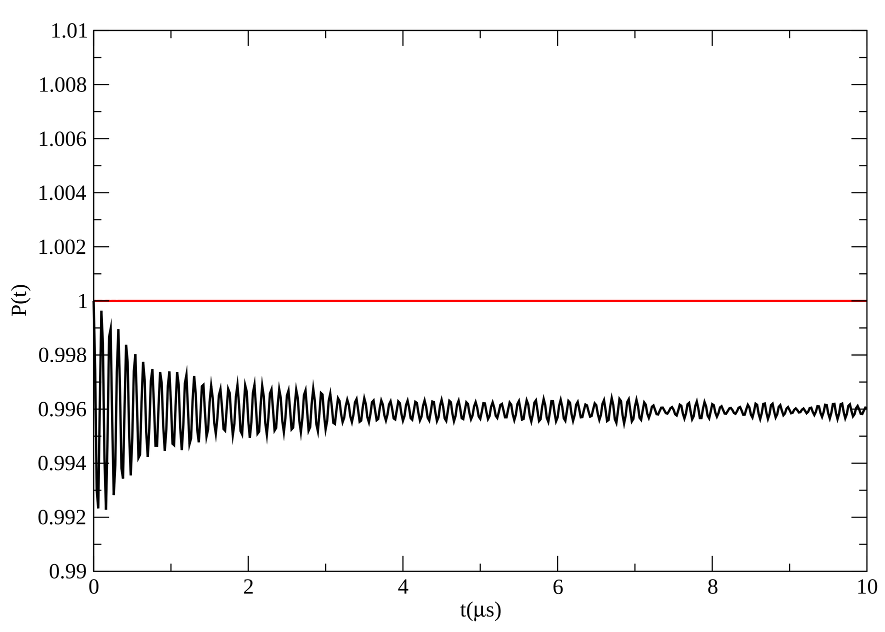

Figure 6: *Simulated μSR data for the initial muon polarization parallel to the magnetic field, which is perpendicular to the film. The polarization is measured along this initial polarization direction. Again black: is for muons stopping near the surface, and red: for muons in the mid-plane.*

4. Conclusion

It will be difficult to directly observe the effects of the transverse fields even for the case where the initial polarization is perpendicular to the film. In Fig. 6 the polarization is very small even for the slice at the edge. The dc offset or the appearance of oscillations could easily occur for slight misalignments of the sample, detectors, muon beam position, or muon polarization direction.

The reduction of the flux per vortex, however, should have interesting consequences. Since the penetration depth, λ, is a function of temperature and the flux per vortex depends on λ, the vortex density must depend on temperature. This in turn will produce changes in the rms field variation over and above that produced just by the variation λ for fixed vortex density. Further, if the vortices become pinned at some temperature, then there will be an internal field change for temperatures below that pinning temperature. Studying the consequences of the reduction of the flux per vortex should be interesting.

References

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